Tutorial 3.1 Coulomb's law for the electric field

From Chapter 3 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

Coulomb's law states that the electric field produced by a point charge q, located at the point $\mathbf{r}' = (x', y', z')$, at an observation point $\mathbf{r} = (x, y, z)$, is

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3},$$

where $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$ is the vector that points from the charge to the observation point and $\mathscr{r}$ is its magnitude. This is an inverse square law because $\boldsymbol{\mathscr{r}}/\mathscr{r}$ is a unit vector in the $\boldsymbol{\mathscr{r}}$ direction; consequently the field falls off as $1/\mathscr{r}^2$.

If several point charges produce the field, then by superposition

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i \boldsymbol{\mathscr{r}}_i}{\mathscr{r}_i^3},$$

where $\boldsymbol{\mathscr{r}}_i = \mathbf{r} - \mathbf{r}_i'$ points from the ith charge to the field point. For a line, surface or volume charge density the sum becomes an integral:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}\,d\ell',$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\mathscr{S}} \frac{\sigma(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}\,da',$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\mathscr{V}} \frac{\rho(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}\,d\tau'.$$

Situation 1

A point charge q is located at $(6, -2, 0)$ m. An observer is located at $(-2, 5, 0)$ m.

(a) On graph paper, draw x and y axes and mark the origin, the charge and the observer. Draw and label the vectors $\mathbf{r}$, $\mathbf{r}'$ and $\boldsymbol{\mathscr{r}}$.



(b) Write out $\mathbf{r}$, $\mathbf{r}'$ and $\boldsymbol{\imath}$ in terms of the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$.

(c) What is the electric field at $\mathbf{r}$?

Situation 2

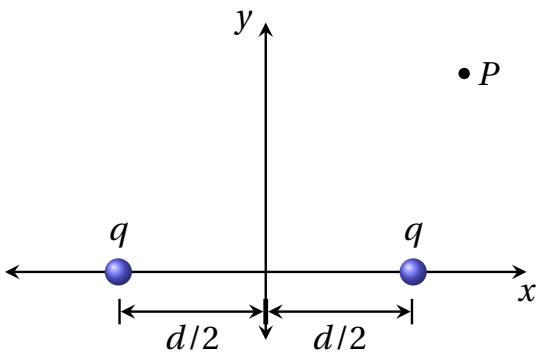


Figure 3.12 Tutorial 3.1, Situation 2: two point charges q on the x axis and an observation point P.

Two point charges, each with charge q, are located on the x axis at the points shown in Figure 3.12. The observer is at an arbitrary location (x, y), marked as point P. The goal is to find the field at P.

(a) Draw the vectors $\mathbf{r}$, $\mathbf{r}'_1$, $\boldsymbol{\imath}_1$, $\mathbf{r}'_2$ and $\boldsymbol{\imath}_2$ on a copy of the figure.

(b) Write out the five vectors in Cartesian components using unit-vector notation, as well as the magnitudes r_1 and r_2.

(c) Write down a general equation for the electric field produced by two point charges. Fill in the equation with the values you have just written out to get the answer.

Situation 3

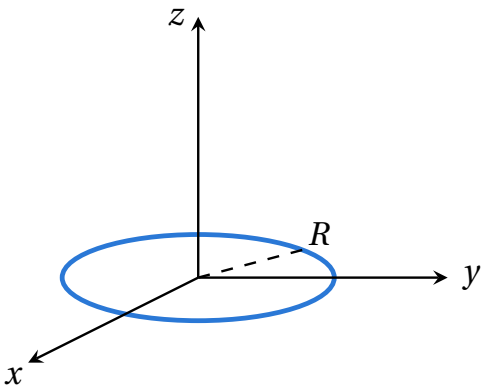


Figure 3.13 Tutorial 3.1, Situation 3: a ring of radius R in the xy plane.

A ring of charge of radius R is centered on the origin and lies in the xy plane, as in Figure 3.13. Its linear charge density is given in cylindrical coordinates by $\lambda = \lambda_0 \cos\phi$. The goal is to find the field at an arbitrary point on the z axis.

(a) Visualize the situation: draw positive and negative charges on the ring to show how and where the charge is located for the given λ. Use your picture to determine by symmetry the direction of the field on the z axis, and explain how you know.

(b) Describe what you have to work with in terms of givens, knowns, unknowns, target variables, and so on.

(c) What will be your starting equation? Briefly describe the underlying principles and concepts at work in that equation.

(d) Mark an arbitrary chunk of charge on the ring, then draw the vectors $\mathbf{r}$, $\mathbf{r}'$ and $\boldsymbol{\imath}$ for that chunk producing a field at an arbitrary point on the z axis. Which of these change during the integral, and how?

(e) Using cylindrical coordinates and cylindrical unit vectors, write down expressions for $\mathbf{r}$, $\mathbf{r}'$, $\boldsymbol{\mathscr{r}}$, $\mathscr{r}$ and $d\ell'$.

(f) Fill in your starting equation with those expressions and the given charge density, then do the integral. Along the way, keep Colton's First Commandment: thou shalt not move a variable unit vector outside the integral. Instead, turn any variable unit vectors into constant ones ($\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$); only then can they come out of the integral. You should find three separate integrals, one for each component, two of which are zero by symmetry.

(g) How can you check whether your answer is reasonable? Consider limiting cases, units, comparison to other problems, and so forth.