Tutorial 11.1 Ampère's law

From Chapter 11 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

The magnetostatics version of Ampère's law, (11.5), says that the circulation of **B** around a closed loop is $\mu_0 I_{\text{enc}}$, where I_{enc} is the amount of current passing through the surface defined by the loop. Just as Gauss's law can be used to determine **E** in certain limited situations of high symmetry, Ampère's law can be used to determine **B** in certain limited situations of high symmetry. The conditions required are quite similar to those for Gauss's law; the table in Section 11.2 sets them side by side. As with Gauss's law, the key is to properly identify the symmetry when the law can be used, and then to choose the correct loop for that symmetry.

Part 1 — Infinite cylinder, current in the z direction

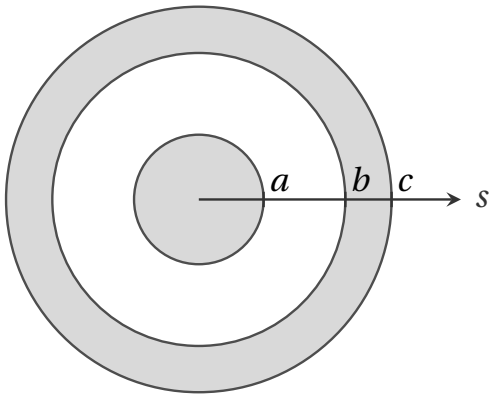


Figure 11.15 Cross section of the coaxial cable of Tutorial 11.1, Part 1. The center conductor ($s < a$) carries a current I_0 out of the page, and the outer conductor ($b < s < c$) carries the return current I_0 into the page.

Figure 11.15 shows the cross section of a long coaxial cable. The center conductor ($s < a$) carries a current I_0 out of the page. The outer conductor ($b < s < c$) carries the return current I_0 into the page. The current density **J** in each of the conductors is constant, although the two are not equal to each other.

(a) First consider each conductor separately. Sketch the center conductor with its current coming out of the page, and its magnetic field. On a second sketch, draw the outer conductor with its current going into the page, and its magnetic field. The total magnetic field will be a superposition of the fields you have drawn.

(b) Now consider the combined situation; sketch both currents on one figure. Why can you use Ampère's law to find **B** in this scenario? What changes to this scenario would make it so that Ampère's law could not be used to find it?

(c) What information are you given about the currents? Use this information to find the current density in the two regions. How does **J** being a vector make applying Ampère's law different from applying Gauss's law?

(d) Sketch an Amperian loop for the region $s < a$, and use it along with Ampère's law to find $\mathbf{B}(s)$ in this region.

(e) Now sketch another Amperian loop for the region $a < s < b$, and use it to find $\mathbf{B}(s)$ in this region.

(f) Sketch a third Amperian loop for the region $b < s < c$, and use it to find $\mathbf{B}(s)$ in this region.

(g) Without doing any calculations, what will $\mathbf{B}(s)$ be in the region $s > c$? Explain by applying Ampère's law.

(h) Sketch the magnitude of $\mathbf{B}$ as a function of s for all four regions. Specifically, consider whether it is (or should be) continuous. Can you use this as a partial check on your work?

Part 2 — Infinite cylinder, current in the ϕ direction

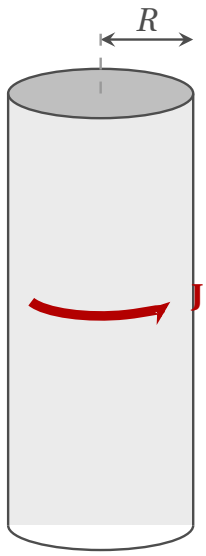


Figure 11.16 The long cylinder of Tutorial 11.1, Part 2, of radius R, with a uniform current density $\mathbf{J} = J_0\hat{\boldsymbol{\phi}}$ circulating around its axis.

For this part, an infinite cylinder of radius R has a constant current density in the ϕ direction, $\mathbf{J} = J_0\hat{\boldsymbol{\phi}}$ (Figure 11.16).

(a) The current here flows in the same direction as in a solenoid. In which ways is this similar to an infinite solenoid, and in which ways is it different?

(b) Which components of the magnetic field are zero, and why? Separately, which coordinates can it depend on, and why?

(c) Make an argument that the field outside the cylinder will be zero.

(d) Select an appropriate Amperian loop to find the field inside the cylinder. Explicitly divide the loop into sections, and make sure that each section has a trivial circulation integral, either because it is obviously zero or else because **B** is parallel to $d\boldsymbol{\ell}$ and constant on that section, which makes the integral $B\ell$. Calculate the current passing through the loop, and solve for **B**.

(e) What changes to the scenario would make this problem not doable with Ampère's law?

Part 3 — Wrap-up

(a) For your answer to 2(d), check to make sure it is reasonable, using at least one of the standard physics methods, such as limiting cases, units, or comparison with other problems.