Tutorial 13.1 B and H in a bar magnet

From Chapter 13 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

Magnetic materials contain bound currents, which produce fields exactly as free currents do (Section 13.3). We chose the definition of the **H** field to give us an Ampère's law that leaves out the bound currents, and that allows an easy calculation of **H** in certain high-symmetry situations:

$$\nabla \times \mathbf{H} = \mathbf{J}_f \qquad \longleftrightarrow \qquad \oint \mathbf{H} \cdot d\boldsymbol{\ell} = I_{f,\text{enc}}.$$

Keep in mind that just because $I_{f,\text{enc}}$ might be zero, that does not necessarily mean that **H** is zero if the situation is not highly symmetric. Also, the divergence of **H** may or may not be zero, unlike the divergence of **B**, which is always zero. The goal of this tutorial is to use the concept of bound currents, boundary conditions, and more, to be able to sketch the **B** and **H** fields in and around a finite bar magnet, and hence to develop more intuition about **B** and **H** fields in general.

Part 1 — Infinite solenoid

(a) Inside an infinite solenoid oriented along the z axis, with a surface current **K** in the $\hat{\boldsymbol{\phi}}$ direction, the magnetic field is constant and points in the z direction. Use Ampère's law to determine the field $\mathbf{B}_{\text{inside}}$ in terms of the surface current.

(b) Use the boundary condition $\mathbf{B}^{\parallel}_{\text{above}} - \mathbf{B}^{\parallel}_{\text{below}} = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}$ to show that the field outside the solenoid must be zero. You needed to know that in order to use Ampère's law properly in (a), but it is still an interesting result when it is reached this way. Hint: which region is "above"? Which way does $\hat{\mathbf{n}}$ point?

Part 2 — Finite solenoid

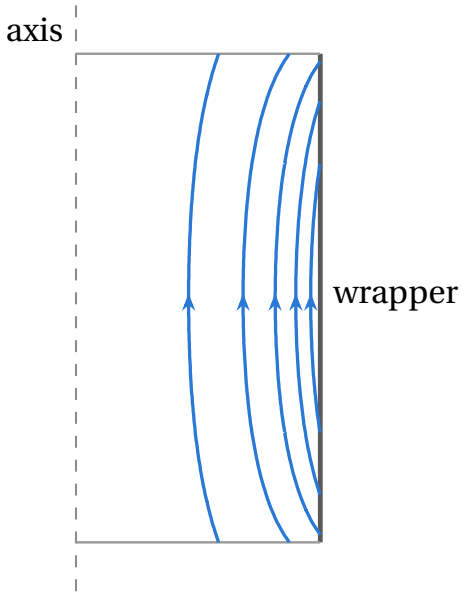


Figure 13.19 The right-hand half of a cross section of a finite solenoid, for Tutorial 13.1, Part 2, with field lines of **B** inside it. Near the ends some of the lines leave through the wrapper.

With a finite solenoid, the magnetic field is no longer constant, but decreases near the top and bottom as the field lines spread out. Therefore the field lines inside the solenoid must look like those in Figure 13.19, drawn for the right-hand half of a cross section. Notice how this requires that some of the field lines leave through the wrapper of the solenoid rather than going out the top and bottom.

(a) Focus on one of the lines that leaves through the right-hand side. Write down the boundary conditions for $B^{\perp}$ and $\mathbf{B}^{\parallel}$ and apply them to this situation to determine the approximate direction of the field just outside the solenoid. You don't need an exact angle for the field line outside the solenoid; we're looking for the general behavior. Hint: consider how strong K is relative to the new B_{inside} of the finite solenoid, which is weaker than the field inside an infinite solenoid. The magnitude of K isn't explicitly known, so just decide if $\mu_0 K$ is larger than, smaller than, or equal to the new B_{inside}, and then decide what that means for the field outside.

(b) Field lines of **B** never begin or end, and for a solenoid they close on themselves. How will this field line close?

(c) Now draw the full picture of the magnetic field of a finite solenoid, with the side loops included.

(d) What will the **H** field look like for this situation? How will the **H** field lines compare to the **B** field lines you have just drawn? Hint: consider the definition of **H**.

(e) What are the boundary conditions for $H^{\perp}$ and $\mathbf{H}^{\parallel}$? Describe how they are consistent with your answer to the previous question.

Part 3 — Finite bar magnet

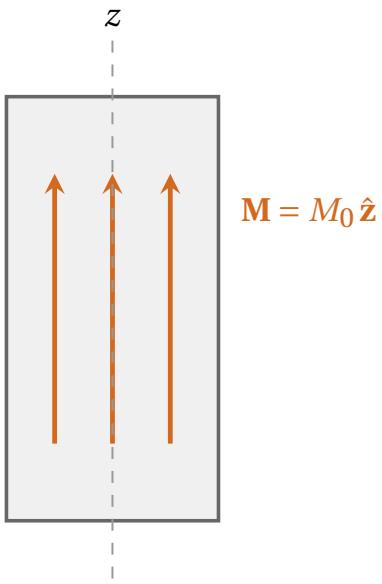


Figure 13.20 The bar magnet of Tutorial 13.1, Part 3, in a cross section through its axis, with its uniform magnetization $\mathbf{M} = M_0\,\hat{\mathbf{z}}$.

A finite cylindrical bar magnet has the magnetization $\mathbf{M} = M_0\,\hat{\mathbf{z}}$ (Figure 13.20). The goal here is to sketch the **B** and **H** field lines and to make sure that everything makes sense in terms of what we know about these fields. To help you do so, first answer all of these questions:

(a) What currents exist? Volume or surface? Free or bound? Where are they, and in what directions do they flow?

(b) What are the boundary conditions for **B** on each surface (top, bottom, and wrapper)?

(c) How does this situation compare with Part 2 with regard to the magnetic field?

(d) What does the **M** field look like when thought of as a vector field in space? Make a field-line sketch of **M**. Does **M** have divergence anywhere? If so, where? Hint: how is divergence seen in field lines?

(e) What is **H** at each point in space? Does **H** have divergence anywhere?

(f) Field lines of **B** never begin or end. Why is that?

(g) Do **H** field lines have to close on themselves? Why or why not?

(h) What are the boundary conditions for **H** on each surface? Be specific, that is, apply them to this particular situation. Hint: the perpendicular boundary condition for **H** comes from the condition $B^{\perp}_{\text{above}} = B^{\perp}_{\text{below}}$.

Now sketch the field lines of **B** and, on a separate drawing, those of **H**, in and around the magnet.

Part 4 — Wrap-up

(a) Check that your sketches are reasonable by considering the limiting case of an infinitely long bar magnet.