Tutorial 16.1 Time-dependent fields

From Chapter 16 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

This chapter and the previous one complete Maxwell's equations by fixing the curl equations to account for time-dependent fields:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

This says that a changing **B** field will create an electric field with curl, and a changing **E** field will create a magnetic field with curl. More than that, mathematically speaking, the changing **B** and **E** fields will create **E** and **B** fields in exactly the same way that a current density **J** creates a magnetic field. In particular, the Amperian-loop technique we have used to find **B** from $\mu_0 \mathbf{J}$ can be used to find **E** from the Faraday current density $\mathbf{J}_{\text{Far}} = -\partial \mathbf{B}/\partial t$, and to find **B** from μ_0 times the displacement current density $\mathbf{J}_D = \varepsilon_0 \partial \mathbf{E}/\partial t$.

Focusing on the second equation for a moment: just as in high symmetry situations we can draw an Amperian loop and use Ampère's law in integral form to find **B** from **J**,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \leftrightarrow \quad \oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{\text{enc}},$$

we can pretend that $\varepsilon_0 \partial \mathbf{E}/\partial t$ is a current density, and find out how much of that type of "current" is enclosed by our Amperian loop:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_D \quad \leftrightarrow \quad \oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{D,\text{enc}}.$$

If you have both regular **J** and also $\mathbf{J}_D$, the loop encloses both, and $\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 (I_{\text{enc}} + I_{D,\text{enc}})$. Similarly, Faraday's law becomes

$$\nabla \times \mathbf{E} = \mathbf{J}_{\text{Far}} \quad \leftrightarrow \quad \oint \mathbf{E} \cdot d\boldsymbol{\ell} = I_{\text{Far,enc}}.$$

The goal of this tutorial is to use the last two equations to calculate induced fields for two situations.

Part 1 — A changing E field creates a B field

The charge of a circular parallel-plate capacitor is increasing according to $q = q_0\left(1 - e^{-t/\tau}\right)$, where q_0 and τ are positive constants. The top plate has the given charge q; the bottom plate has charge $-q$. The plates have radius R and are separated in the z direction by a distance d, where $R \gg d$, so you can neglect edge effects. What magnetic field is produced in between the capacitor plates?

(a) Draw a picture or diagram to help you visualize what is going on.

(b) Why is there a magnetic field? Describe in words.

(c) What will the displacement current be (magnitude, direction)? Make a rough sketch of the magnetic field that curls around it on your picture above.

(d) Now, getting ready to solve the problem, describe what you have to work with in terms of givens, knowns, unknowns, target variables, and so forth.

(e) Come up with a plan to use the integral form of the Ampère–Maxwell law, with the displacement current, to solve this problem, similar to how you have used Ampère's law in the past, and carry out that plan.

Part 2 — A changing B field creates an E field

The current in a solenoid (n turns per length) is increasing according to $I = I_0\left(1 - e^{-t/\tau}\right)$, where I_0 and τ are positive constants. The solenoid's length is much longer than its radius, so you can neglect edge effects. What electric field is produced inside the solenoid?

(a) Do this by drawing a picture, finding the Faraday current, drawing an appropriate Amperian loop, and using Faraday's law in the form $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = I_{\text{Far,enc}}$ to solve for $\mathbf{E}$.

Part 3 — Wrap-up

(a) Are there any ways you can think of to consider whether the answers you obtained in 1(e) and 2(a) are reasonable? (Limiting cases, units, comparison to other problems, and so forth.)