Tutorial 21.1 Fresnel equations for s-polarization

From Chapter 21 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

Section 21.7.2 derived the Fresnel equations for p-polarized light, whose electric field lies in the plane of incidence. In this tutorial you derive them for s-polarized light, whose electric field is perpendicular to the plane of incidence, by the same steps. Use the axes of that section: the boundary is the plane $z = 0$, the plane of incidence is the xz plane, and $\hat{\mathbf{y}}$ points out of the page. The incident wave travels in material 1 at the angle θ_I from the normal, the reflected wave at $\theta_R = \theta_I$, and the transmitted wave at the angle θ_T given by Snell's law (21.19). Use α and β as defined in (21.20) and (21.15).

Part 1 — The fields

Figure 21.23 shows the electric field of each wave pointing out of the page, along $+\hat{\mathbf{y}}$.

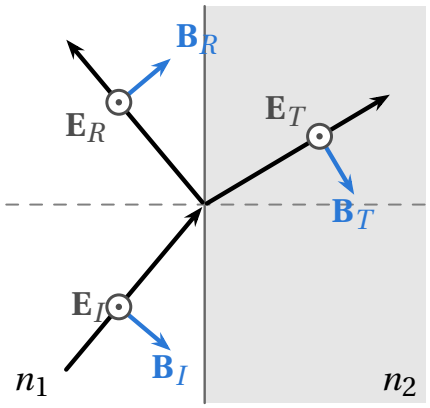


Figure 21.23 The setup of Tutorial 21.1: s-polarized light, with the electric fields of all three waves out of the page and the magnetic fields (blue) in the plane of incidence.

If one of them really points into the page, its amplitude will come out negative.

(a) Why do the magnetic fields of the incident and transmitted waves point down, toward $-x$, while the magnetic field of the reflected wave points up?

(b) Write the incident, reflected, and transmitted electric and magnetic fields in complex exponential form. For each wave give the wave vector in terms of its angle, the direction of each field as its own unit vector, and the amplitude of **B** in terms of the amplitude of **E**.

Part 2 — The amplitudes

(a) Impose the boundary conditions on $\mathbf{E}^{\parallel}$ and on $\mathbf{B}^{\parallel}/\mu$ at $z = 0$ to get two equations that involve the three electric field amplitudes. What happens to the exponential factors? Why does the condition on $\varepsilon E^{\perp}$ give nothing here?

(b) Solve the two equations for the ratios of the amplitudes, $r_s = \tilde{E}_{0R}/\tilde{E}_{0I}$ and $t_s = \tilde{E}_{0T}/\tilde{E}_{0I}$, in terms of α and β.

(c) Find the reflectance R_s and the transmittance T_s. (Remember the factor that the cross sections of the beams contributed for p-polarization.) Show that $R_s + T_s = 1$.

Part 3 — Wrap-up

Figure 21.24 shows r_s, t_s, R_s, and T_s for light going from air into glass with $n = 1.50$.

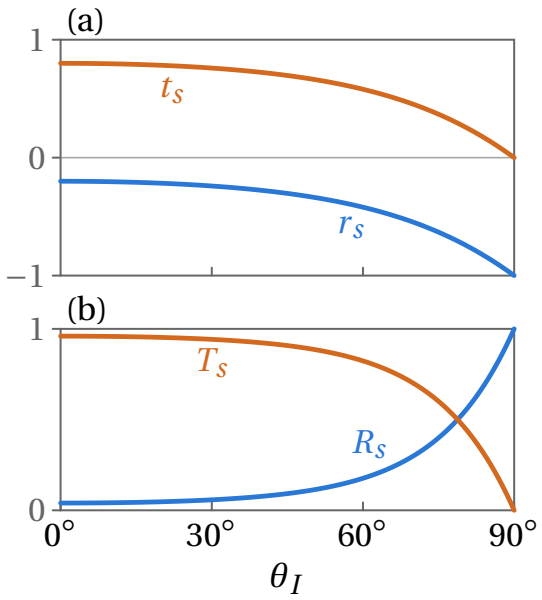


Figure 21.24 The Fresnel equations for s-polarization, for light going from air into glass with $n = 1.50$: (a) r_s and t_s; (b) R_s and T_s, against the angle of incidence. Compare Figure 21.22.

(a) Check your r_s and t_s at normal incidence against (21.16), and show that their grazing incidence behavior matches Figure 21.24.

(b) For light going from air into glass with $n = 1.50$ at 45°, find R_s, and compare it with R_p from (21.22). Which polarization does a glass surface reflect more strongly?