Tutorial 22.2 A metal across the spectrum

From Chapter 22 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

In this tutorial you use the Drude model (22.24) to follow one metal, sodium, from microwaves to the ultraviolet. Sodium has one conduction electron per atom, 2.5×10^{28} per cubic meter, and its resistivity at 20°C is $4.77 \times 10^{-8}\,\Omega\,\mathrm{m}$ (Table 19.2). Keep enough digits in each complex number to tell which of its parts is small. Where $\tilde{\varepsilon}_r$ is nearly real and negative, $n = \varepsilon_r''/2\kappa$ is more accurate on a calculator than the roots (22.7).

Part 1 — The parameters

(a) Find the plasma frequency ω_p of sodium and the vacuum wavelength $2\pi c/\omega_p$. In what part of the spectrum is it?

(b) Use the connection between (22.24) and the conductivity of Section 22.4.2 to find the damping rate Γ from the resistivity. How many times smaller than ω_p is it?

Part 2 — Microwaves

(a) For microwaves at 10 GHz, compare ω with Γ, and simplify (22.24) accordingly. Find $\tilde{\varepsilon}_r$. Which of its parts is larger, and by how much?

(b) Find $\tilde{n}$, and the distance in which the amplitude of the wave falls by the factor $1/e$. Compare that distance with the skin depth (22.9) computed with the DC conductivity.

(c) Find the reflectance at normal incidence from air.

Part 3 — Visible light

(a) For light with $\lambda_0 = 500\,\text{nm}$, compare ω with Γ and with ω_p, simplify (22.24), and find $\tilde{\varepsilon}_r$.

(b) Find $\tilde{n}$, the reflectance, and the distance in which the amplitude falls by $1/e$.

Part 4 — Ultraviolet

(a) Repeat Part 3 for $\lambda_0 = 150\,\text{nm}$. Is n larger or smaller than 1?

(b) How far does this light travel in sodium before its intensity falls by $1/e$? Could a film of sodium $1\,\mu\text{m}$ thick transmit much of it? Could it transmit the light of Part 3?

Part 5 — Wrap-up

(a) Using your results, explain in a few sentences why sodium looks like a shiny metal.

(b) At what wavelength does the model predict that sodium becomes transparent? Compare it with Wood's measurement (Section 22.5.3).