Tutorial 22.1 Total internal reflection, s-polarized

From Chapter 22 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

Section 22.1 found the reflected wave and the evanescent wave for p-polarized light beyond the critical angle. In this tutorial you do the same for s-polarized light, whose electric field is perpendicular to the plane of incidence. Use the axes and the notation of that section and of Tutorial 21.1: the boundary is the plane $z = 0$, material 1 (index n_1) is at $z < 0$ and material 2 (index $n_2 < n_1$) is at $z > 0$, the plane of incidence is the xz plane, both materials have the same permeability, and $\theta_I > \theta_c$. You will need your coefficients r_s and t_s from Tutorial 21.1. Beyond the critical angle they become the complex numbers $\tilde{r}_s$ and $\tilde{t}_s$.

Part 1 — The reflected wave

(a) Beyond the critical angle $\tilde{\alpha} = i|\tilde{\alpha}|$ is purely imaginary, while β is real. Use this in your r_s to show that $|\tilde{r}_s| = 1$. What fraction of the incident power is reflected?

(b) Find the phase ϕ_s of $\tilde{r}_s$ as a formula, and evaluate it for glass with $n = 1.50$ into air at $\theta_I = 45°$. Compare it with the phase ϕ_p of Example 22.1.

Part 2 — The evanescent wave

(a) Write the transmitted electric and magnetic fields in complex exponential form. Start from your fields of Tutorial 21.1, and replace $\sin\theta_T$ and $\cos\theta_T$ by their values beyond the critical angle, in terms of k_x, κ, and k_2.

(b) Take the real parts. Choose the zero of time so that $\tilde{E}_{0T}$ is real. Which component of **B** is in phase with **E**, and which is 90° out of phase with it?

(c) Check that your fields satisfy all four of Maxwell's equations in material 2.

Part 3 — The energy

(a) Find the average Poynting vector of the evanescent wave. Show that on average no energy crosses the boundary, and find the direction in which energy does flow.

Part 4 — Wrap-up

(a) Compare your results with those for p-polarization in Section 22.1. What is the same, and what differs?

(b) $\tilde{t}_s$ is not zero, yet no power is transmitted. Explain how both can be true.