Tutorial 26.2 Power through a plane

From Chapter 26 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

This tutorial uses the fields of a point charge moving at constant velocity, (26.23). In them **R** is the vector that points from the *present* location of the charge to the point where you want to know the fields, **v** is the charge's velocity, and θ is the angle between **v** and **R**. Parts 1 and 2 also use the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$ (Section 20.2).

Part 1 — General

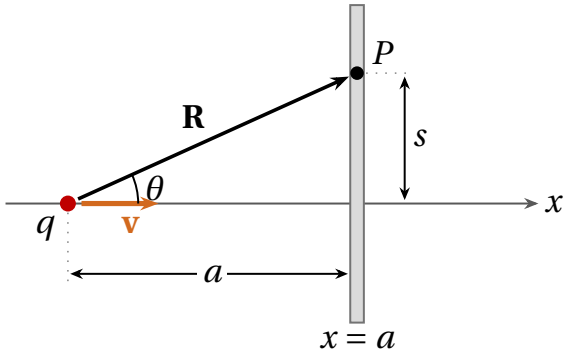


Figure 26.11 Tutorial 26.2: a point charge q, now at the origin, moving at constant velocity **v** toward the plane $x = a$, seen edge on, and an arbitrary point in the plane, a distance s from the x axis.

Figure 26.11 shows a point charge q traveling at constant velocity **v** in the x direction, currently at the origin and moving toward a plane at $x = a$.

(a) What will the electric and magnetic fields surrounding the point charge look like in general, and in what directions will they point at the arbitrary point marked in the figure? Draw the fields on a copy of the figure.

(b) In what direction will the Poynting vector point at various locations on the plane? Think about the arbitrary point first, and then about the rest of the plane. Sketch it.

(c) What significance does your answer to (b) have? In other words, what does the Poynting vector represent physically, and what does it mean for the Poynting vector field to look that way?

(d) Repeat (b) and (c) for a plane located behind the charge, for example at $x = -a$.

Part 2 — Finding the power passing through the plane

(a) The goal of the rest of this tutorial is to find the electromagnetic power passing through the infinite plane at $x = a$. Write down your knowns and unknowns, and describe a "game plan" for how you could use what you have been given to find it. What are the steps along the way from what you know to what you want to know? Avoid looking at the following steps when you make your plan; it is fine if your plan is not the way the tutorial does things.

(b) One piece of your game plan likely involves the electric field on the plane. Find it now at the arbitrary point, in terms of v, s, a, and $\hat{\mathbf{R}}$.

(c) Another piece may involve the magnetic field on the plane. We can bypass that step, however, if we are clever. Consider that $\mathbf{E} \times \mathbf{B} = (1/c^2)\,\mathbf{E} \times (\mathbf{v} \times \mathbf{E})$. What about this suggests that there might be a simpler way to find $\mathbf{S}$ than finding $\mathbf{B}$? Why is it important to consider things like this when choosing the steps toward solving a problem?

(d) Derive an expression for the Poynting vector in terms of the electric field alone (no $\mathbf{B}$), the velocity $\mathbf{v}$, and the angle θ. (Hint: the "BAC-CAB" vector identity should help.)

(e) The electromagnetic power passing through the plane is $\int_{\text{plane}} \mathbf{S} \cdot d\mathbf{a}$. Why? Which component of $\mathbf{S}$ matters here? Find that component, in terms of the magnitudes of $\mathbf{E}$ and $\mathbf{v}$ and the angle θ.

(f) Complete the problem: do the integral $\int_{\text{plane}} \mathbf{S} \cdot d\mathbf{a}$, and find how much power passes through the plane.

Part 3 — Wrap-up

(a) Think of the standard ways that physicists determine whether an answer is reasonable: limiting cases, dimensional analysis, comparison with other problems, and so forth. Apply at least one of them to your answer.

(b) What are the implications of your answer to Part 2(f)?