Tutorial 28.1 Multipole radiation

From Chapter 28 of *Physics of Electricity and Magnetism*, by John S. Colton (free at `https://em.byu.edu`).

Name: _____

Background

Section 28.4 found the first three terms of the radiation of a small source: the powers (28.26) of electric dipole, magnetic dipole, and electric quadrupole radiation at each instant, and their averages (28.27) for moments that oscillate as $\cos\omega t$. The electric and magnetic dipole terms each have the angular dependence $\sin^2\theta$, where θ is measured from $\ddot{\mathbf{p}}$ or $\ddot{\mathbf{m}}$. In this tutorial you compare the sizes of the three terms, and then you apply them to charges that move in circles.

Part 1 — Relative magnitudes

First, compare the sizes of the three kinds of radiation. Picture a set of charges and currents that change in time but return to their original configuration every period $T = 2\pi/\omega$. Suppose that the electric dipole, the magnetic dipole, and the electric quadrupole all have the same characteristic length d, time T, and charge q. A *characteristic* length, time, or charge is a rough size for that quantity: d is about the size of the source, T is the period, and q is about how much charge moves. Answers in terms of these scales are estimates that show how a quantity depends on d, T, and q; for example, the charges move at speeds of about d/T.

(a) In terms of these characteristic scales, what is the electric dipole moment? (Recall that $\mathbf{p} = \sum_k q_k \mathbf{r}_k$.) What is the average power of the electric dipole radiation?

(b) In terms of the characteristic scales, what is the magnetic dipole moment? (Recall that $\mathbf{m} = I\mathbf{a}$, where in terms of the characteristic scales the current is a charge divided by a time and the area is a length squared.) What is the average power of the magnetic dipole radiation?

(c) Now compare. Use your answers to (a) and (b) to find $\langle P_{\mathrm{M1}}\rangle / \langle P_{\mathrm{E1}}\rangle$, and put it in terms of the characteristic speed $v = d/T$. Under what conditions is the magnetic dipole radiation much weaker than the electric dipole radiation?

(d) In terms of the characteristic scales, what is the quadrupole moment? (Just give one representative component, such as Q_{xy}. Recall that for point charges $Q_{ij} = \sum_k q_k(\frac{3}{2} r_{k,i} r_{k,j} - \frac{1}{2} r_k^2 \delta_{ij})$, from Section 8.5.) Roughly what is the average power of the electric quadrupole radiation?

(e) Use your answers to (a) and (d) to find $\langle P_{\mathrm{E2}}\rangle / \langle P_{\mathrm{E1}}\rangle$, also in terms of $v = d/T$. When is it small?

(f) What do your last two answers say about the relative sizes of P_{M1} and P_{E2}, and about how to use the multipole expansion of radiation? For comparison, in the multipole expansion of the electric potential (Section 8.3) the terms come in the order monopole, dipole, quadrupole, and so on, and far away you can generally stop at the first one that is not zero.

Part 2 — A point charge moving in a circle

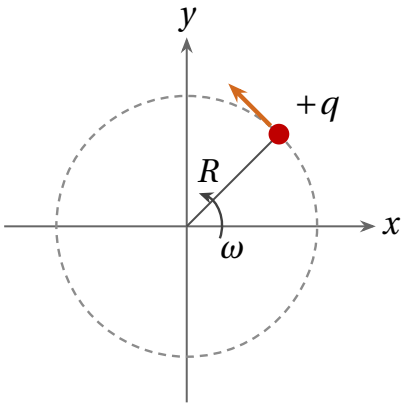


Figure 28.17 Tutorial 28.1, Part 2: a charge q moving counterclockwise around a circle of radius R in the xy plane.

Now consider three arrangements of charges that move in circles. First, a single charge q moves around a circle of radius R at the constant speed v, with $\omega = v/R$, so that its position is $\mathbf{r}' = R\cos\omega t\,\hat{\mathbf{x}} + R\sin\omega t\,\hat{\mathbf{y}}$ (Figure 28.17).

(a) Why might it be useful to describe the radiation of charges that move in circles with the multipole expansion, rather than, say, with Larmor's formula for each charge?

(b) Find the electric dipole moment $\mathbf{p}$ of the charge. Does it have a harmonic time dependence?

(c) Find the magnetic dipole moment **m** of the charge. Take its current to be q divided by the period of its motion. Does **m** have a harmonic time dependence?

(d) Does this system emit electric dipole radiation? If so, how much power? Does it emit magnetic dipole radiation? If so, how much? Which kind dominates?

Part 3 — Two opposite charges on a rigid rotor

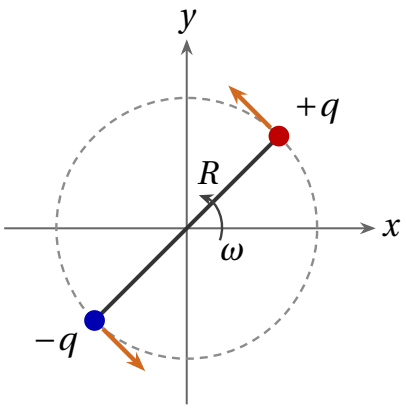


Figure 28.18 Tutorial 28.1, Part 3: charges $+q$ and $-q$ at the ends of a rigid rod of length $2R$ that rotates about its center at the angular velocity ω.

Now consider two opposite charges, $+q$ and $-q$, fixed to the ends of a rigid rod of length $2R$ that rotates about its center at the angular velocity ω (Figure 28.18). This arrangement is called a rigid rotor.

(a) Find the electric dipole moment. Does it have a harmonic time dependence?

(b) Find the magnetic dipole moment. Does it have a harmonic time dependence?

(c) Does this system emit electric dipole radiation? If so, how much? Does it emit magnetic dipole radiation? If so, how much? Which kind dominates?

Part 4 — Two equal charges on a rigid rotor

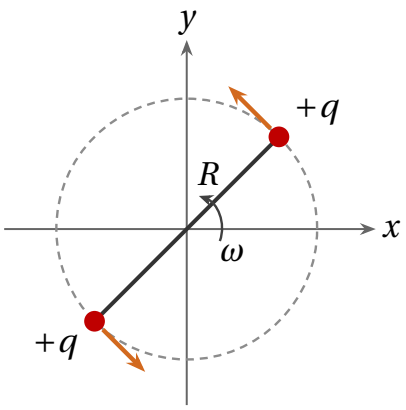


Figure 28.19 Tutorial 28.1, Part 4: two charges $+q$ at the ends of the rotor of Part 3.

Now put two equal charges, both $+q$, on the ends of the rotor (Figure 28.19).

(a) Find the electric dipole moment. Does it have a harmonic time dependence?

(b) Find the magnetic dipole moment. Does it have a harmonic time dependence?

(c) Does this system emit electric dipole radiation? If so, how much? Does it emit magnetic dipole radiation? If so, how much? Which kind dominates?

Part 5 — Wrap-up

(a) Thinking about what happens physically, explain the difference between your answers to Parts 3 and 4.

(b) Think of the standard ways that physicists determine whether an answer is reasonable: limiting cases, dimensional analysis, comparison with other problems, and so forth. Apply at least one of them to one of your answers.